Lesson outline

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Key words:
Solar system
Galilean moons
dwarf planets
remote planet weighing
exoplanets

Solar System. Weighing planets.

Topic:	Planets and smaller bodies in the Solar System. Searching for extrasolar planets. Weighing planets remotely.
Students' age:	15-19
Time:	3 lessons

Subjects:
geography
physics

LESSONS IDEA

Interdisciplinary classes about the planets and smaller bodies in the Solar System, during which students learn about:

- physical and chemical data of planets, dwarf planets, asteroids, comets, and interplanetary dust;
- space probes sent to explore these worlds;
- the search methods for extrasolar planets;
- how to observe the Galilean moons of Jupiter;
- how to determine the mass of Jupiter based on measurements of the motion of its satellites.

The teacher discusses the multimedia presentation about the Solar System bodies (attached to the scenario). The teacher draws attention to the size scale of the central star and planets, as well as the differences in the size of the planets themselves (rocky planets versus gas giants). It can be also useful to mention the names of space probes that have been sent in their direction, e.g.

- Mercury - Mariner 2, BepiColompo
- Venus - Venera, Pioneer Venus 1, Magellan
- Mars - Viking, Pathfinder, Sojourner, Spirit, Opportunity, Curiosity, Perseverance, InSight
- Jupiter - Pioneer 10, Pioneer 11, Voyager 1, Voyager 2, Galileo, Juno
- Saturn - Pioneer 11, Voyager 1, Cassini-Huygens
- Uranus and Neptune - Voyager 2
- Pluto - New Horizons
- Ceres - Dawn
- Comets - Vega, Giotto, Deep Space 1, Stardust, Rosetta-Philae
- Planetoids - Galileo, HEAR Shoemaker, Hayabusa, Rosetta-Philae, New Horizons
- Sun - SOHO, STEREO, TRACE, Solar Orbiter, Ulysses

- The Solar System is one of billions of planetary systems scattered across the space. It is our home, the place where planet Earth along with all other celestial bodies have been orbiting for billions of years. The research of the solar family of planets is one of the most exciting tasks that astronomers and engineers who design space probes and samplers are facing.
- Nowadays we are also aware of thousands of exoplanets orbiting distant stars. Thanks to the discoveries of ground-based telescopes (e.g. OGLE project) and space telescopes (including Hubble, Kepler, TESS), their number is constantly growing. Transit, radial velocities, and gravitational microlensing methods are used to discover exoplanets. Only a few extrasolar planets may be observed directly. In the future the new generation of ground-based and space telescopes will allow us to observe exoplanets directly and study the conditions in their respective atmospheres. Perhaps signs of extraterrestrial life will be found on of them.

- The Italian scholar Galileo Galilei, using a telescope on January 7th, 1610, initially discovered three moons of Jupiter: Io, Europa, Callisto; and on January 11th, 1610, he discovered one more moon: Ganymede. Initially he thought that he had discovered three fixed stars. Two of them were on one side, the third on the other side of Jupiter. On January 8th, all three objects were on one side of Jupiter, and Galileo thought it was the planet that had shifted from its previous position. The next night was cloudy and no observations could be made. On January 10th and 11th he observed only two stars located on one side of the planet. He concluded from that the third one was covered by Jupiter and was aligned with it. He described the result of this observation in the words: 'There seem to be three moving stars around Jupiter, which no one has seen before'. On January 13th and 15th Galileo had already simultaneously observed four small objects near Jupiter. Based on these observations, Galileo realized that these objects were actually orbiting Jupiter. The discovery of Jupiter's moons became an argument for the heliocentric theory, providing compelling evidence that the universe is constructed in a different way than the geocentric theory presented.

Students can observe Jupiter's moons the night before or a couple of nights before the classes - if the weather and planetary visibility is favourable - paying attention to their alignment. The observations can be performed using just a usual binoculars. The Stellarium program may assist in identification. To analyze and calculate Jupiter's mass, a series of systematic observations are needed (preferably night after night) to cover a 1 or 2 weeks interval. It can also be 1 or 2 months if usual atmospheric conditions are poor. These can be either photographs showing the moons (the planet's disk will usually be overexposed; try a simple digital processing of the image to get an effect similar to the one presented here; you can also make one reference exposure with the same equipment with a shorter time, to be able to determine the size of the planet's disk and the scale of the image) or sketches, but in the latter case it is important to have an exact reproduction of the image, preserving the scale. The pictures allow you to measure the distance to the moons from the center of the planet's disk (the unit may be either radius or equatorial diameter). Their identification depends on the regularity of the photo sample - if the series is relatively long and complete, it can be done by ourselves, otherwise it is advised to use the sky simulator – Stellarium ⇒link

The goal is to prepare a chart of the distance to the moons from the center of the planet's disk against time (similar to the one shown below) and to read the period of the change (the circulation T) and the amplitude (the maximum distance a). If possible, we make such a chart for each of the four moons, so that under favorable circumstances a similar set of T and a data for all of them will be obtained.

An Excel spreadsheet can be used to prepare the graph (see the attached excel file).

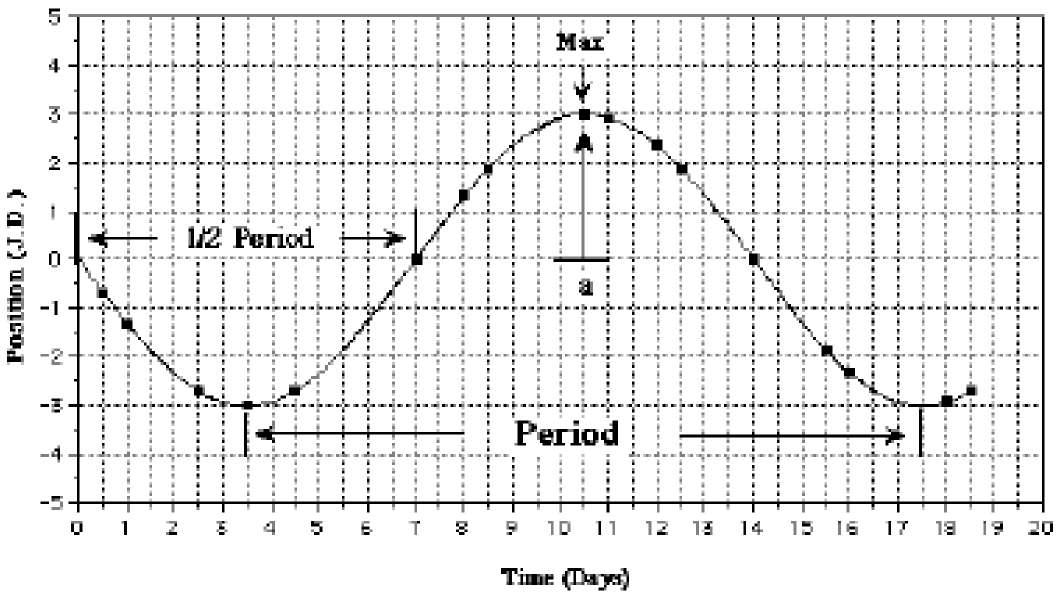
✍ The collected material allows to calculate the mass of Jupiter and in general and also any other celestial body that is orbited by satellites. The data obtained from the graphs will be used for final calculations in the next lesson.

The teacher shows how to calculate the mass of Jupiter using Kepler's third law of planetary motion and the formulas for gravitational force and centripetal force. Probably the simplest way to calculate the mass of Jupiter will be the method suggested on here ⇒link

In general, the relation between the period T and the distance a, known as the law of motion of planets is presented in the form: a^3 / T^2 = const. It is worth remembering two important comments on this formulated law. Firstly, with respect to planets, taking the Earth for reference and assuming its values T = 1 year, a = 1 a.u. (astronomical unit) - in a very simple way it is possible to determine the distance on the basis of the position, e.g., for a'= 4 a.u. we get T' = 8 years (since $a'^3 = 4^3 = 64$, so in view of $T'^2 = 64$ we determine T' = 8); similarly, from the known period of the planet's orbit we determine its distance, e.g., knowing that Jupiter orbits the Sun in 11.86 years we determine its distance as 5.2 a.u.

Secondly, the mass of the central object, and in the case of planets - the Sun, is hidden in the constant.

A simple comparison of gravitational force to centripetal force (and substituting for velocity



the formula $2\pi a/T$) brings us $M = 4\pi^2 a^3/(GT^2)$, where G is the gravity constant. Students should be asked to see what value did they get if they substitute the data for Earth in SI units.

Similarly by observing the motion of any satellite and determining its distance, the mass of a planet, such as Earth, may be measured.

What is more, since Kepler's third law is an universal relation, we can calculate the ratio a^3 / T^2 on the basis of observations of any object, determine how many times greater or smaller the mass of the central object is, if we use the most convenient units in the considered reference system. We can formulate the orbital period of Jupiter's moons in years and their distance in a.u. to determine the mass of Jupiter relative to the mass of the Sun, but this is not very convenient. It would be much more suitable to determine the period of T's circulation in 'lunar months' (i.e., by dividing by 27.32 days) and the distance a - in 'lunar distances' (this is, of course, the Earth's moon, so the determined distance of Jupiter's Moon is divided by 384400 km) - the mass of Jupiter relative to the mass of the Earth is calculated. For example, for Callisto T = 16.689 days and a = 1882700 km, so in 'lunar units' respectively T = 16.689/27.32 = 0.611 and a = 1882700/384400 = 4.898. We calculate a^3 / T^2 in these units (the mass relative to the mass of the Earth is determined):

$$a^3 / T^2 = (4,898)^3 / (0,611)^2 = 314,75$$

If the calculations are correct, we can get an independent result from the observations of each of the four moons and then calculate the mean value of Jupiter.

Please note (for verification purposes) that by using the same method, calculating the Earth's motion around the Sun: T=365,25/27,32 = 13,369 and a = 149600000/384400 = 389,2 we have $a^3 / T^2 = (389,2)^3 / (13,369)^2 = 329853$. This is the number determining the ratio of the Sun's mass to the Earth's mass - in fact, taking from the tables of Sun and Earth masses: 1.98855 ×1030 / 5.97219×1024 = 332968, which of course, within rounding limits, is the same result! More importantly, however, historically speaking, the first method is closer to the original method - measurement by observation; as the first researchers could not read the mass of the Sun from tables because such tables have not existed yet!

- The presented method of remotely weighing celestial bodies shows the power of physics and mathematics used by a perceptive observer. What is important, the applied mathematical apparatus is not beyond the knowledge and abilities of a high school student. This rather simple exercise gives great satisfaction to young researchers.
- The use of the Stellarium simulator allows measurements to be made in the classroom, practically at any time and date. However, if the weather is favourable enough, it is highly recommended to collect the materials for calculations on your own with a telescope.

ADDITIONAL MATERIALS

- How to calculate the mass of Jupiter ⇒link
- Stellarium online ⇒link

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